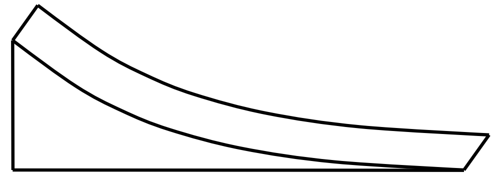
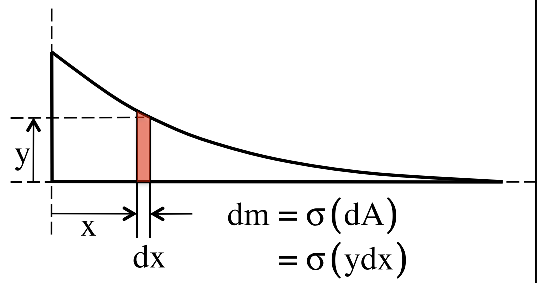
Problem 12.5

Determine the *x-component* of the *center of gravity* of the object shown to the right.



We need to define a coordinate axis, which I've done in the sketch to the right. We also need to identify all the mass associated with an arbitrary *x-coordinate* comprised in a differential section "dx." That is also shown in the sketch. Noting that the total mass will be the sum of the differential masses:



$$dm = \sigma(dA) = \sigma(y\,dx) = \sigma\left(\frac{(x-3)^2}{9}\right)dx$$

$$x_{cg} = \frac{\int_{x=0}^{L} x\,dm}{\int_{x=0}^{L} dm} = \frac{\int_{x=0}^{L} x(\cancel{\sigma}y\,dx)}{\int_{x=0}^{L} (\cancel{\sigma}y\,dx)}$$

$$x_{cg} = \frac{\int_{x=0}^{L} x\left(\left(\frac{(x-3)^2}{\cancel{9}}\right)dx\right)}{\int_{x=0}^{L}\left(\left(\frac{(x-3)^2}{\cancel{9}}\right)dx\right)}$$

$$= \frac{\int_{x=0}^{L}(x^3 - 6x^2 + 9x)\,dx}{\int_{x=0}^{L}(x^2 - 6x + 9)\,dx}$$

$$= \frac{\left(\frac{x^4}{4} - 2x^3 + \frac{9}{2}x^2\right)\Big|_{x=0}^{L}}{\left(\frac{x^3}{3} - 3x^2 + 9x\right)\Big|_{x=0}^{L}} = \frac{\left(\frac{L^4}{4} - 2L^3 + \frac{9}{2}L^2\right)}{\left(\frac{L^3}{3} - 3L^2 + 9L\right)} = \frac{\left(\frac{L^3}{4} - 2L^2 + \frac{9}{2}L\right)}{\left(\frac{L^2}{3} - 3L + 9\right)}$$

$$= \frac{\left(\frac{(3.00\text{ m})^3}{4} - 2(3.00\text{ m})^2 + \frac{9}{2}(3.00\text{ m})\right)}{\left(\frac{(3.00\text{ m})^2}{3} - 3(3.00\text{ m}) + 9.00\right)}$$

$$= .750\text{ m}$$